Econ 802

First Midterm Exam

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All questions have equal weight. It is a good idea to read the entire exam before you start writing, and work first on the questions where you feel most confident.

1. Assume we have a perfectly competitive firm that maximizes profit. Provide a clear and detailed response to each of the following statements. Use math, graphs, or words as needed.

(a) "If the firm has a single output and all of the input requirement sets V(y) are convex, then the firm's production possibility set Y must also be convex."

(b) "If the price of an output rises then the quantity of that output must rise, and if the price of an input rises then the quantity of that input must fall."

(c) "The firm chooses an output price equal to its marginal cost."

2. Consider the Cobb-Douglas production function $y = x_1^{\alpha} x_2^{\beta}$ where $x_1 \geq 0$, $x_2 \geq 0$, $\alpha > 0$ and $\beta > 0$. Output price is $p > 0$ and input prices are $w_1 > 0$, $w_2 > 0$.

(a) Does the profit maximization problem always have a solution? If it does have a solution, can you be confident that the solution is unique? Explain.

(b) Does the cost minimization problem always have a solution? If it does have a solution, can you be confident that the solution is unique? Explain.

(c) Assume that the firm has constant returns to scale and the cost minimization problem has a solution. Calculate the cost shares $w_1x_1 / (w_1x_1 + w_2x_2)$ and $w_2x_2 / (w_1x_1 + w_2x_2)$ for an arbitrary output $y > 0$. Then give an economic interpretation of your results.

3. Acme Inc. has the production function $y = \max \{Ax_1 + Bx_2; Cx_1 + Dx_2\}$ where A, B, C, D are all positive constants with $A > C$ and $D > B$. The firm must obey the non-negativity constraints $x_1 \geq 0$ and $x_2 \geq 0$.

(a) Draw a graph in (x_1, x_2) space showing the set of points where $y = Ax_1 + Bx_2$ and the set of points where $y = Cx_1 + Dx_2$. Briefly explain your reasoning.

(b) Draw a typical isoquant Q(y). Indicate the value of each intercept and the slopes of any line segments. Is the input requirement set V(y) convex? Is V(y) strictly convex? Justify your answers.

(c) Give a complete description of the conditional input demand functions $x_1(w, y)$ and $x_2(w, y)$. Indicate the circumstances under which the firm chooses boundary or interior solutions, and say whether the solutions are unique.

4. A firm has two inputs and one output. In the short run $x_1 \geq 0$ is variable and $x_2 > 0$ is fixed. The firm's short run profit function is

$$\pi(p, w_1, w_2, x_2) = x_2[(1-\alpha)p^{1/(1-\alpha)}(\alpha/w_1)^{\alpha/(1-\alpha)} - w_2]$$

(a) Compute the input demand function $x_i(p, w_1, w_2, x_2)$. Does this function have the comparative static properties you would expect? Explain.

(b) Compute the output supply function $y(p, w_1, w_2, x_2)$. Does this function have the comparative static properties you would expect? Explain.

(c) Suppose x_2 is variable in the long run. Sometimes the long run profit function is well defined and sometimes it is not. Discuss this issue carefully.

5. Here are some miscellaneous questions.

(a) A firm has one input $y_1 \leq 0$ and one output $y_2 \geq 0$. The prices are $p_1 > 0$ and $p_2 > 0$. The firm is observed in two time periods t = 1, 2 (use superscripts for time periods). Draw a graph in (y_1, y_2) space showing some observations for periods t = 1, 2 that <u>violate</u> the Weak Axiom of Profit Maximization. Explain.

(b) Let $p = (p_1 .. p_n) > 0$ be a price vector and let $y = (y_1 .. y_n)$ be a production plan, where outputs are positive and inputs are negative. Let Y be the set of feasible production plans and let $\pi(p)$ be the profit function. Suppose $y^* \in Y$ has $py^* \geq py$ for all $y \in Y$. Prove mathematically that $\pi(tp) = t\pi(p)$ for all $t > 0$ and then explain verbally why this makes sense.

(c) A firm has a production function $y = f(x)$ where $x = (x_1 .. x_n) \geq 0$ are the inputs. The output price is $p > 0$ and the input prices are $w = (w_1 .. w_n) > 0$. Suppose that $x^* > 0$ satisfies both the first order and <u>sufficient</u> second order conditions for the profit maximization problem. Your friend says that x^* will also satisfy both the first order and <u>sufficient</u> second order conditions for the cost minimization problem when the firm must produce output $y^* = f(x^*)$. Is this true, false, or uncertain? Explain carefully.